

UTM - DIRECT Map Projections (Snyder)

0.1 Notations and Definitions (as per USGS Professional Paper 1395)

0.1.1 Required Input:

φ	Latitude of point (positive north) Comment (B) Note: Angles in radians
λ	Longitude of point (positive east) Comment (L) Note: Western longs negative
ω	Rectifying latitude
φ_0	Latitude of grid origin Comment (Bs) Note: Northern lats positive
λ_0	Longitude of Central Meridian (=Grid Origin) Comment (Lo)
E_f	False Easting of the Grid Origin Comment for everywhere = 500,000 [m]
N_f	False Northing of the Grid Origin Comment for north lats=0, for south lats=10,000,000[m]
S	Meridional distance to point (on ellipsoid)[m]
S_0	Meridional distance along CM on projection plane, from equator to φ_0 Comment [m]
k_0	Grid scale factor assigned to the CM (sets secant depth, =09996 always)
k	Grid scale factor at the point
a	Ellipsoid semi-major axis Comment [m]
f	Flattening of the ellipsoid
Z	Desired projection zone ID Comment Note: Used as look-up index

0.1.2 Intermediate calculated values

e^2 First eccentricity of the ellipsoid

e'^2 Second eccentricity of the ellipsoid

0.1.3 Desired Output

N Northing of point **Comment** [m]

E Easting of point **Comment** [m]

γ Convergency angle at point **Comment** (C)

0.2 Calculate Zone Constants

$$e^2 = \sqrt{2f - f^2}$$

$$e'^2 = e^2 / (1 - e^2)$$

Z = (enter zone #, 1-60)

$\varphi_0 = 0$ (for northern lats = 0, for southern = 10,000,000)

$\lambda_0 = 6Z - 183$ (longitude of central meridian)

$k_0 = 0.9996$ (from 1/2,500)

0.3 Calculate Direct Conversion Computation

$$\begin{aligned}
 m1 &= (1 - e^2/4 - 3e^4/64 - 5e^6/256) \varphi \\
 m2 &= -(3e^2/8 + 3e^4/32 + 45e^6/1024) \sin(2\varphi) \\
 m3 &= (15e^4/256 + 45e^6/1024) \sin(4\varphi) \\
 m4 &= -(35e^6/3072) \sin(6\varphi) \\
 M &= a(m1 + m2 + m3 + m4)
 \end{aligned}$$

$$\begin{aligned}
 m1o &= (1 - e^2/4 - 3e^4/64 - 5e^6/256) \varphi_0 \\
 m2o &= -(3e^2/8 + 3e^4/32 + 45e^6/1024) \sin(2\varphi_0) \\
 m3o &= (15e^4/256 + 45e^6/1024) \sin(4\varphi_0) \\
 m4o &= -(35e^6/3072) \sin(6\varphi_0)
 \end{aligned}$$

$$M_o = a(m1 + m2 + m3 + m4) \quad \text{Comment } M_o = 0 \text{ for all UTM, because } \varphi_0 = 0$$

$$W = \sqrt{1 - e^2 \sin^2(\varphi)}$$

$$N = a/W$$

$$T = \tan^2(\varphi)$$

$$C = e'^2 \cos^2(\varphi)$$

$$A = (\lambda - \lambda_o) \cos(\varphi) \quad \text{Comment } \text{For western longs, } \lambda \text{ has to be negative}$$

$$m5 = M - M_o + N \tan(\varphi) (A^2/2 + (5 - T + 9C + 4C^2)A^4/24 + (61 - 58T + T^2 + 600C - 330e'^2)A^6/720)$$

$$m6 = A + (1 - T + C)A^3/6 + (5 - 18T + T^2 + 72C - 58e'^2)A^5/120$$

$$m7 = 1 + (1 + C)A^2/2 + (5 - 4T + 42C + 13C^2 - 28e'^2)A^4/24 + (61 - 148T + 16T^2)A^6/720$$

$$\text{Northing} = Nf + k_0 m5$$

$$\text{Easting} = Ef + k_0 Nm6$$

$$k = k_0 * m7$$

$$\gamma = (\lambda - \lambda_o) \sin(\varphi)$$