Lambert Conformal Conic Map Projection - INVERSE

0.1 Notations and Definitions (Source: NOAA Manual NOS NGS 5 - State Plane Coordinate System of 1983 - James E. Stem - v.1990, page 38+)

0.1.1 Required Input:

N Northing of point Comment [m]

E Easting of point Comment [m]

Z Desired projection zone ID Comment Note: Used as look-up index

0.1.2 Variables:

φ_s Latitude of Standard Parallel South Comment (Bs)

φ_n Latitude of Standard Parallel North Comment (Bn)

φ_b Latitude of Grid Origin Comment (Bb)

λ_o ....Longitude of Central Meridian (=Grid Origin) Comment (Lo)

k_o Scale factor at the Central Parallel Comment Note: sets secant depth

N_f False Northing of the Grid Origin Comment Note: For NAD83, in [meters]

E_f False Easting of the Grid Origin Comment [m]

a Ellipsoid semi-major axis Comment [m]

f Flattening of the ellipsoid

0.1.3 Intermediate calculated values

φ_o	Central parallel of projection Comment (Bo)
N_o	Northing of Central Parallel
R	Mapping Radius at latitude (φ)
R_b	Mapping Radius at latitude (φ_b)
R_o	Mapping Radius at latitude (φ_o)
K	Mapping Radius at Equator
Q...	Isometric Latitude
e	First eccentricity of the ellipsoid

0.1.4 Desired Output

φ	Latitude of point (positive north) Comment (B) Note: angles in radians
λ	Longitude of point (negative west) Comment (L)
k	Grid scale factor at point
γ	Convergency angle at point Comment (C)

0.2 Calculate Zone Constants

$$e = \sqrt{2 * f - f^2}$$

$$\varphi_0 = (\varphi_s - \varphi_n)/2$$

$$Q_s = \frac{1}{2}\left[ln\frac{1 + sin(\varphi_s)}{1 - sin(\varphi_s)} - e * ln\frac{1 + e * sin(\varphi_s)}{1 - e * sin(\varphi_s)}\right]$$

$$W_s = \sqrt{1 - e^2 * sin^2(\varphi_S)}$$

$$Q_n = \frac{1}{2}\left[ln\frac{1 + sin(\varphi_n)}{1 - sin(\varphi_n)} - e * ln\frac{1 + e * sin(\varphi_n)}{1 - e * sin(\varphi_n)}\right]$$

$$W_n = \sqrt{1 - e^2 * sin^2(\varphi_n)}$$

$$.......Q_b = \frac{1}{2}\left[ln\frac{1 + sin(\varphi_b)}{1 - sin(\varphi_b)} - e * ln\frac{1 + e * sin(\varphi_b)}{1 - e * sin(\varphi_b)}\right]$$

$$W_b = \sqrt{1 - e^2 * sin^2(\varphi_b)}$$

$$\sin(\varphi_o) = \frac{ln\left[W_n * cos(\varphi_s) / (W_s * cos(\varphi_n))\right]}{Q_n - Q_s}$$

$$Q_0 = \frac{1}{2}\left[ln\frac{1 + sin(\varphi_0)}{1 - sin(\varphi_0)} - e * ln\frac{1 + e * sin(\varphi_0)}{1 - e * sin(\varphi_0)}\right]$$

$$W_o = \sqrt{1 - e^2 * sin^2(\varphi_o)}$$

$$.......K = \frac{a * cos(\varphi_s) * exp(Q_s * sin(\varphi_0))}{W_s * sin(\varphi_0)}$$

$$R_b = K/\exp(Q_b * sin(\varphi_o))$$

$$R_o = K/\exp(Q_o * sin(\varphi_o))$$

$$k_o = (W_o * R_o * tan(\varphi_o))/a$$

$$N_o = R_b + N_f - R_o$$

0.3 Calculate Inverse Coversion Computation

$R_b = K / \exp(Q_b * sin(\varphi_o))$

$R' = R_b - N + N_f$

$E' = E - E_f$

$\gamma = \arctan(E' / R')$ # <-------

$\lambda = \lambda_0 - \gamma / sin(\varphi_0)$ # <-------

$R = \sqrt{(R'^2 + E'^2)}$

$Q = [\ln(K/R)] / \sin(\varphi_0)$

$\sin(\varphi) = \frac{\exp(2*Q) - 1}{\exp(2*Q) + 1}$ #Seed value for iteration

$f1 = \frac{1}{2} [\ln\left(\frac{1+\sin(\varphi)}{1-\sin(\varphi)}\right) - e* \ln\left(\frac{1+e*\sin(\varphi)}{1-e*\sin(\varphi)}\right)] - Q$

$f2 = 1/(1-\sin^2(\varphi)) - e^2/(1 - e^2 * \sin^2(\varphi))$

$\sin(\varphi) = \sin(\varphi) - (f_1/f_2)$ #Repeat 4 times

$\varphi = \arcsin[\sin(\varphi)]$ # <-------

$W = \sqrt{1 - e^2 * sin^2(\varphi)}$

$k = W*R*\sin(\varphi_o) / (a * cos(\varphi))$ # <-------