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## Lambert Conformal Conic Map Projection - DIRECT

### 0.1 Notations and Definitions (Source: NOAA Manual NOS NGS 5 - State Plane Coordinate System of 1983 - James E. Stem - v.1990, page 36+)

#### 0.1.1 Required Input:

| | | |
|---|---|---|
| $\varphi$ | Latitude of point (positive north) Comment | (B) Note: Enter all angles in radians |
| $\lambda$ | Longitude of point (positive east) Comment | (L) Note: Western longitudes are negative |
| $\varphi_s$ | Latitude of Standard Parallel South Comment | (Bs) Note: Northern latitudes are positive |
| $\varphi_n$ | Latitude of Standard Parallel North Comment | (Bn) |
| $\varphi_b$ | Latitude of Grid Origin Comment | (Bb) |
| $\lambda_o$ | Longitude of Central Meridian (=Grid Origin) Comment | (Lo) |
| $k_o$ | Scale factor at the Central Parallel Comment | Note: This scale factor sets the secant depth |
| $N_f$ | False Northing of the Grid Origin Comment | Note: For NAD83, in [meters] |
| $E_f$ | False Easting of the Grid Origin Comment | [m] |
| a | Ellipsoid semi-major axis Comment | [m] |
| f | Flattening of the ellipsoid | |
| Z | Desired projection zone ID Comment | Note: Used as look-up index |

### 0.1.2 Intermediate calculated values

| | |
|---|---|
| $\varphi_o$ | Central parallel of projection Comment (Bo) |
| $N_o$ | Northing of Central Parallel |
| R | Mapping Radius at latitude ( $\varphi$ ) |
| $R_b$ | Mapping Radius at latitude ( $\varphi_b$ ) |
| $R_o$ | Mapping Radius at latitude ( $\varphi_o$ ) |
| K | Mapping Radius at Equator |
| Q... | Isometric Latitude |
| e | First eccentricity of the ellipsoid |

### 0.1.3 Desired Output

| | |
|---|---|
| N | Northing of point Comment [m] |
| E | Easting of point Comment [m] |
| k | Grid scale factor at point |
| $\gamma$ | Convergency angle at point Comment (C) |

## 0.2 Calculate Zone Constants

$$e = \sqrt{2 * f - f^2}$$

$$Q_s = \frac{1}{2}\left[ln\frac{1 + sin(\varphi_s)}{1 - sin(\varphi_s)} - e * ln\frac{1 + e * sin(\varphi_s)}{1 - e * sin(\varphi_s)}\right]$$

$$W_s = \sqrt{1 - e^2 * sin^2(\varphi_S)}$$

$$Q_n = \frac{1}{2}\left[ln\frac{1 + sin(\varphi_n)}{1 - sin(\varphi_n)} - e * ln\frac{1 + e * sin(\varphi_n)}{1 - e * sin(\varphi_n)}\right]$$

$$W_n = \sqrt{1 - e^2 * sin^2(\varphi_n)}$$

$$Q_b = \frac{1}{2}\left[ln\frac{1 + sin(\varphi_b)}{1 - sin(\varphi_b)} - e * ln\frac{1 + e * sin(\varphi_b)}{1 - e * sin(\varphi_b)}\right]$$

$$W_b = \sqrt{1 - e^2 * sin^2(\varphi_b)}$$

$$sin(\varphi_o) = \frac{ln\left[W_n * cos(\varphi_s) / (W_s * cos(\varphi_n))\right]}{Q_n - Q_s}$$

$$Q_0 = \frac{1}{2}\left[ln\frac{1 + sin(\varphi_0)}{1 - sin(\varphi_0)} - e * ln\frac{1 + e * sin(\varphi_0)}{1 - e * sin(\varphi_0)}\right]$$

$$W_o = \sqrt{1 - e^2 * sin^2(\varphi_o)}$$

$$K = \frac{a * cos(\varphi_s) * exp(Q_s * sin(\varphi_0))}{W_s * sin(\varphi_0)}$$

$$R_b = K / \exp(Q_b * sin(\varphi_o))$$

$$R_o = K / \exp(Q_o * sin(\varphi_o))$$

$$k_o = (W_o * R_o * tan(\varphi_o)) / a$$

$$N_o = R_b + N_f - R_o$$

## 0.3 Calculate Direct Coversion Computation

$$Q = \left[ ln\frac{1+sin(\varphi)}{1-sin(\varphi)} - e * ln\frac{1+e*sin(\varphi)}{1-e*sin(\varphi)} \right] / 2$$

$$W = \sqrt{1 - e^2 * sin^2(\varphi)}$$

$$R = K / \exp\left(Q * sin(\varphi_o)\right)$$

$$\gamma = (\lambda_o - \lambda) * sin(\varphi_o)$$

$$N = R_b + N_b - R * cos(\gamma)$$

$$E = E_o + R * sin(\gamma)$$

$$k = W*R*sin(\varphi_o) / (a * cos(\varphi))$$